# **French horn diversions**

#### Introduction

Playing the French horn I found some properties of the instrument very interesting. Scanning internet and literature I spotted fascinating explanations and because that might be of interest to other nosy players it is presented here. The French horn and its playing practice have been empirically developed, as it is with most musical instruments. Instrument designers and top musicians from past ages gave their persistence, musicality and patience for the development of the modern French horn and playing technique <sup>1-4</sup>. Mid-19<sup>th</sup> century Hermann von Helmholtz performed the first extended theoretical investigations <sup>5)</sup> on music and sound as well as on the functioning of musical instruments. Later on that research has been continued and the models verified with experimental measurements <sup>6, 7)</sup>. It was found that for explaining the functioning and properties of the instruments to some extent, rather complicated physical models were required. With the simple models used here below only primary properties can be explained.

### Acoustics

In fact sound is a temporal variation of atmospheric pressure. It moves through air with the speed of sound (about 340 meters per second) and in open air the propagation is equally distributed in all directions. When sound moves through a (narrow) tube the propagation direction is guided by the tube wall. When that sound arrives at the (open) tube end, the propagation is no longer guided by the tube wall and it travels in all directions again. So a fraction of the sound will travel back into the tube (sort of reflection). Forward traveling and back traveling sound in the tube will interact then. The resulting local sound intensity depends on the amount of acoustic energy that enters the tube (from the mouthpiece), the fraction of reflection at the ends of the tube and the internal losses in the tube. Now suppose that the sound is a tone, a steady repeating variation of the local pressure with a time period of  $t_p$  (and a frequency  $f_t$  of the tone, where  $f_t = 1 / t_p$ ). There will be a special



case when halve the periodical time  $t_p$  equals the travel time of sound trough the tube, see in top of the figure alongside. In that case back and forth traveling sound enhance each other and that is called resonance. Inside the tube a so called standing wave will appear because maxima and minima of pressure variations have a fixed position inside the tube. The wavelength  $\lambda_t$  of the tone is proportional to the speed of sound  $v_{air}$  and inversely proportional to the frequency  $f_t$  or:  $\lambda_t = v_{air} / f_t$ . The same situation will appear when one complete wavelength fits inside the tube, or three halves etc. etc. In the figure the first four possibilities for resonance are depicted for an open tube. A French horn is a brass wind instrument where mouthpiece, a long tube bent in a loop with at the end a bell form together an acoustic resonator. The lips of the player are placed in front of the mouthpiece and forced to vibrate by blowing a thin flow of air in between the lips. When the frequency of the lip vibrations and the length of the tube have a proper ratio, resonance will occur resulting in a sonorous tone. The frequency of tones that can be played in this way (natural overtones) are multiples (harmonics) of the lowest (pedal) tone  $f_1$ , so  $f_2=2f_1$ ,  $f_3=3f_1$ ,  $f_4=4f_1$  etc., see the table below for a horn with pedal tone F. A skilled horn player can easily play up to the sixteenth overtone (compare flageolets at a violin). Playing a horn, the lips are the source of vibration and the resonance of the air column in the tube enhances the sound intensity, playing a violin the string is the source of vibration which is amplified by the resonance of the body structure of the violin.



Not using any valve the horn has a tone range of about four octaves. That is large a range but does not include all tones of the scale. In between the first octave  $(F_1-F_2)$  there are no intermediary tones at all. Inside the second octave  $(F_2-F_3)$  there is only a just fifth (131 Hz instead of 130,8 Hz). In between the third octave  $(F_3-F_4)$  there is a just third a' (218 Hz instead of 220 Hz), the just fifth c' (262 Hz instead of 261,6 Hz) and a seventh that is very off-key (306 Hz). In the fourth octave there are seven tones of which the greater part are not on key and thus very false (see the large tables below).

And it becomes worse. The end of the horn is bell-shaped (diameter up to 35 cm) so that more sound can be obtained than from just a narrow tube. Apparently that has consequences for the properties of the resonator. The wavelength of the  $16^{th}$  overtone of an F horn is  $\pm 0.5$  m, the wavelength of the lowest tone  $\pm 8$  m. For the low tones it will not matter too much when the diameter of the tube becomes a little wider towards the end. But the high notes which have a much shorter wavelength will face a shorter tube. The wavelength of these tones almost fits crosswise in the bell and that will give insufficient guiding for a definite propagation direction of the sound inside the tube. That results in a rise of the resonance frequencies for the high notes. In particular the highest octave will be a little stretched in this way, destroying the correct tuning of these resonances.

#### The right hand

Horn players in the classical era had already empirically discovered that the tone pitch can be influenced by placing the hand in the bell. Therefor the horn was held at the loop with the left hand and the right hand (the most active one) was kept inside the bell. Later on valves have been invented and mounted at the left hand side of the horn to allow the right hand to continue its usual task. For the right hand there are three things to do: balancing the horn, optimizing the acoustical properties in the bell and the fine tuning of certain resonance frequencies, see <sup>1-4</sup>. In the seventies of the  $20^{\text{th}}$  century the American scientist John Backus performed systematic measurements on the influence of the right hand in the bell <sup>7</sup> and in 2009 Adam Watts presented his dissertation on this subject <sup>8</sup>. The two illustrations below are from his work.



Conn 8D Open F-Side |Zin| Comparison for Hand Placement Adam Watts 7/2/09

The graphs here above describe the acoustical impedance of an F horn at different frequencies for two conditions: the blue graph has been recorded with open bell and for the red

graph the left hand was placed inside the bell as shown in the picture alongside. The higher the graph reaches, the easier a tone can be produced at that frequency. Comparing these two graphs shows that: 1) with the hand in the bell the distance between the resonance peaks is fairly constant where without the hand the distance of the



peaks becomes bigger and irregular at higher frequencies: 2) with the hand in the bell the resonance peaks become higher and steeper, in particular at the higher frequencies. With the hand at the proper position the resonances are at almost exact proper (theoretical) frequency positions and the higher notes are much easier to play.

The ideal position of the right hand is dependent on the shape of hand and bell. That ideal position must be established every time when playing a new horn: use an electronic tuning device, play the different harmonics (use no valves) in the middle of their resonances as much as possible and try different positions of the hand until the octaves and fifths are pure. In that same position the hand must have maximum freedom for stopping and tuning of individual tones. To do that the hand is bended or folded into the bell, fingers stay in place (fingers and thumb together with fingernails against the bell wall). Once the ideal position has been found, there is never need to change that.

Attention should be paid to use a neutral embouchure when investigating the ideal hand position. When you play a certain horn over a long time you become used to automatically correct the tuning of the different tones of that horn. That should be avoided in this case.

Insertion of the right hand into the bell extends the effective length of the resonator for higher harmonics because the inside bell diameter is decreased in this way. The purpose of the bell was to enhance the sound output of the tube and that is gradually obstructed by the right hand again. That is an example of the compromises over a centuries long period that resulted in the beautiful sound the horn can produce nowadays.

### The left hand

To increase the number of performable notes on the horn valves have been added. Pressing a valve extends the length of the resonator by a certain amount. Most commonly three valves are used; the first valve adds a tube length corresponding to a pitch lowering of a major second, the second valve a minor second and the third valve a minor third. In this way an F horn with the first valve actuated becomes an E-flat horn, with the second valve an E horn and with the third valve a D horn. In the figure below the resonance frequencies are given for an F horn with three valves. These are theoretical values assuming perfect position of the right hand making pure octaves etc. The tube lengths at the valves have been chosen to match exactly a major and minor second and minor third lowering of the resonance frequencies for the respective valves.



In this way a lot of performable tones have been added to the horn but in the high range there are still many tones that does not properly fit within our tone scale. Next to that the simultaneous use of two or three valves subsequently leads to resonance frequencies that are too high (see the lower three series in the figure). For an F-horn (resonator length about 3.8 m) the length that is added (a minor second =  $\sqrt[12]{2} = 5,95$  %) by 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> valve is respectively 46,6 cm, 22,6 cm and 71,9 cm. But when the first valve has already been actuated, for the lowering of another minor second not 22.6 cm is required but 5,95 % of (380 + 46,6 cm) or 25,4 cm. That is the reason why a combination of valves will always result in sharp tones.

Another way to have more well-tuned tones available is the combining of complete horns of different length. An often used combination is an assembly of an F and a B-flat horn, selected by the player using a fourth valve. These two horns share the mouthpiece (with lead pipe) and bell and in between the two different horns can be chosen. Here below an overview of performable tones on a B-flat horn is given.



Over the whole tone range of F and B-flat horn the performable tones are nice complementary divided over the tone scale. This combining of horns extends the possibilities for well-tuned resonances throughout the playing range. For instance the sounding C (130,81, 261,63 en 523,25 Hz) can be played much better in tune on an F horn (130,95, 261, 90 en 523,80 Hz) than on a B-flat horn (133,26, 259,54 en 524,43 Hz), whereas other resonances are better in tune on a B-flat horn (see the tables below).

The resonator of an alphorn has a (tree!) length of several meters and is conical over the full length, i.e. the inside diameter of the tube increases from the mouthpiece (about 0.5 cm) steadily to



the bell of about 10 cm, so that is much less than the 35 cm of a French horn bell. When an alphorn is equipped with the properly adapted mouthpiece the different overtones are better tuned as compared with a French horn without right hand in position. The inside diameter of a French horn resonator increases steadily the first 40-50 cm after the

mouthpiece throat, is then constant (cylindrical) in the valve section, then increases slowly towards the bell. That is certainly not an ideal resonator but the trick with the right hand helps a lot already. And modern horn makers in the meantime have found possibilities for optimizing the tuning of notoriously off-scale resonances. Still a set of well-trained ears is essential in proper French horn playing.

On the next two pages tables are presented of theoretical resonance frequencies of F and Bflat horn as if the right hand is in ideal position.

Frequencies (Hz) of equal tempered tones next to resonance frequencies of the F horn										
Note in C	Note in F	Frequency (Hz)	F horn natural	F horn valve 2	F horn valve 1	F horn valve 3	F horn valve 1+2	F horn valve 2+3	F horn valve 1+3	F horn valve 1+2+3
B <sub>0</sub>	F#	30,87								31,83
C <sub>1</sub>	G	32,70							33,27	
C <sub>1#</sub>	G#	34,65						34,95		
D1	Α	36,71				36,71	36,92			
D <sub>1#</sub>	A <sub>#</sub>	38,89			38,89					
E <sub>1</sub>	В	41,20		41,20						
F <sub>1</sub>	С	43,65	43,65							
B <sub>1</sub>	F#	61,74								63,68
C <sub>2</sub>	G	65,41							66,55	
C <sub>2#</sub>	G <sub>#</sub>	69,30						69,90		
D <sub>2</sub>	Α	73,42				73,42	73,85			
D <sub>2#</sub>	A <sub>#</sub>	77,78			77,78					
E <sub>2</sub>	В	82,41		82,41						
F <sub>2</sub>	С	87,31	87,31							
F <sub>2#</sub>	C#	92,50								95,49
G <sub>2</sub>	D	98,00							99,80	
G <sub>2#</sub>	D#	103,83						104,90		
A <sub>2</sub>	E	110,00			116.70	110,00	110,80			
A <sub>2#</sub>	F	110,54		100.00	116,70					107.00
B <sub>2</sub>	F#	123,47	120.05	123,00					122.10	127,30
C3	G	120,61	130,95					120.90	155,10	
D	G#	1/6 92				146.92	147.70	139,80		
D <sub>3</sub>	A	140,00			155.56	140,65	147,70			150.20
D <sub>3#</sub>	R#	164.91		164.91	155,50				166.40	139,20
E3	C	174.61	174.61	104,01				174.80	100,40	
Far	C.	185.00	174,01			183.60	184.60	174,00		191.00
1 3# Ga	D	196.00			194 40	105,00	104,00		199.60	171,00
Ga#	D	207.65		206.00	191,10			209 73	199,00	
A2	E	220.00	218.25	200,00		220.25	221.50	200,10		222.80
A2#	F	233.08			233,30				232.90	,
B <sub>3</sub>	F#	246.94		247.20				244,70		254.60
C <sub>4</sub>	G	261,63	261,90	-		257,00	258,50		266,20	
C <sub>4#</sub>	G#	277,18			272,20			279,60		286,50
D <sub>4</sub>	A	293,66		288,40		293,66	295,40		299,50	
D <sub>4#</sub>	A <sub>#</sub>	311,13	305,55		311,13			314,60		318,30
E4	В	329,63		329,63		330,00	332,40		332,70	
F <sub>4</sub>	С	349,23	349,23		350,00			349,50		350,20
F4#	C#	369,99		370,80		367,00	369,20		366,10	382,00
G <sub>4</sub>	D	392,00	392,90		389,00			384,50	399,20	
G <sub>4#</sub>	D <sub>#</sub>	415,30		412,00		404,00	406,20	419,40		413,80
A <sub>4</sub>	E	440,00	436,50		427,80	440,60	443,10		432,60	445,60
A <sub>4#</sub>	F	466,16		453,20	467,00			454,40	465,80	477,80
B <sub>4</sub>	F#	493,88	480,20	494,40		477,00	480,00	489,30	499,20	509,30
C <sub>5</sub>	G	523,25	523,80		505,50	514,00	516,90	524,30	532,40	
C <sub>5#</sub>	G#	554,37		535 <b>,60</b>	544,00	550,70	552 <b>,9</b> 0	55 <b>9,30</b>		
D <sub>5</sub>	А	587,33	567,50	57 <b>6,8</b> 0	583,30	587,37	590,80			
D <sub>5#</sub>	A <sub>#</sub>	622,25	611,10	618,00	622,25					
E <sub>5</sub>	В	659,25	654,80	<b>659,</b> 25						
F <sub>5</sub>	С	698,46	698,46							

Frequencies (Hz) of equal tempered tones next to resonance frequencies of the B flat horn										
Note in C	Note in F	Frequency (Hz)	B flat horn natural	B flat horn valve 2	B flat horn valve 1	B flat horn valve 3	B flat horn valve 1+2	B flat horn valve 2+3	B flat horn valve 1+3	B flat horn valve 1+2+3
E <sub>1</sub>	В	41,20								42,49
F <sub>1</sub>	С	43,65							44,42	
F <sub>1#</sub>	C <sub>#</sub>	46,25						46,65		
G1	D	49,00				49,00	49,30			
G <sub>1#</sub>	D <sub>#</sub>	51,91			51,91					
A <sub>1</sub>	E	55,00	50.07	55,00						
A <sub>1#</sub>	1	58,27	58,27							
B <sub>1</sub>	r <sub>#</sub>	01,/4								
C <sub>2</sub>	G	69.30								
D <sub>2</sub> #	Δ	73 42								
Do#	A.,	77.78								
E <sub>2</sub>	B	82,41								84,98
F <sub>2</sub>	С	87,31							88,84	
F <sub>2#</sub>	C <sub>#</sub>	92,50						93,33		
G <sub>2</sub>	D	98,00				98,00	98,59			
G <sub>2#</sub>	D#	103,83			103,83					
A <sub>2</sub>	E	110,00		110,00						
A <sub>2#</sub>	F	116,54	11 <b>6</b> ,54							
B <sub>2</sub>	F <sub>#</sub>	123,47								127,48
C <sub>3</sub>	G	130,81							133,26	
C <sub>3#</sub>	G <sub>#</sub>	138,59						139,99		
D <sub>3</sub>	A	146,83				147,00	147,88			
D <sub>3#</sub>	A <sub>#</sub>	155,56		16100	155,75					1 (0.07
E <sub>3</sub>	в	104,81	174.01	164,98					177.60	169,97
Г <sub>3</sub> Е	C C	1/4,01	1/4,81					196.65	1//,09	
г <sub>3#</sub> С-	С <sub>#</sub>	196.00				196.00	197 19	180,05		
Gau	D	207.65			207.65	190,00	197,19			212.46
A <sub>2</sub>	E	220,00		220.00	207,00				222.11	212,10
A2#	F	233.08	233.08					233.31	;	
B <sub>3</sub>	F#	246,94				245,00	246,49			254,83
C <sub>4</sub>	G	261,63			259,54				266,53	-
C <sub>4#</sub>	G <sub>#</sub>	277,18		274,97				279,98		
D <sub>4</sub>	А	293,66	291,35			294,00	295,79			297,30
D <sub>4#</sub>	A <sub>#</sub>	311,13			311,45				310,95	
E <sub>4</sub>	В	329,63		329,97				326,64		339,77
F <sub>4</sub>	С	349,23	349,62			343,00	345,08		355,37	
F <sub>4#</sub>	C <sub>#</sub>	369,99			363,36			373,30		382,41
G <sub>4</sub>	D	392,00		384,96		392,00	394,35		399,79	
G <sub>4#</sub>	D <sub>#</sub>	415,30	407,89		415,30			419,96		424,72
A <sub>4</sub>	E	440,00	100.10	440,00	162.12	441,00	443,68	100.00	444,21	1.67.10
A <sub>4#</sub>	F	466,16	466,16	40.4.05	467,17	400.00	402.00	466,63	400.64	467,42
	r <sub>#</sub>	493,88	524.42	494,95	510.02	490,00	492,98	512.20	488,04	209,91
C <sub>5</sub>	G	554.37	524,45	540.05	517,00	539.01	542.22	559.96	555,00	552.40
~5# D∈	A A	587 33	582 70	2,27	571.00	588.01	591.58	559,90	577 49	594.89
D:#	A	622.25	562,70	604 95	622.90	500,01	0,1,00	606 62	621 91	637 39
E5	B	659.25	640.98	659.94	022,70	636.78	640.83	653.28	666.33	679.88
F.5	с	698,46	699.25		674.57	686.01	690.12	699.95	710.75	
F <sub>5#</sub>	C <sub>#</sub>	739,99		714,94	726,72	735,01	739,41	746,61		
G <sub>5</sub>	D	783,99	757,52	769,93	778,63	783,99	788,71			
G <sub>5#</sub>	D <sub>#</sub>	830,61	815,79	824,93	830,61					
A <sub>5</sub>	E	880,00	874,06	880,00						
A <sub>5#</sub>	F	932,33	932,33							

#### The tuning slides

At distinct positions in the resonator sliding tubes have been incorporated. That is useful for draining of condense water and, by sliding in and out, tuning the exact length of the resonator. There is one main tuning slide for the horn (in case of a double horn two) and also for every valve there is a separate tuning slide. In this way the length of resonator and the tube extensions for the valves can accurately be tuned.

When tuning a horn at first the main slide (no valves) is tuned to have the F (respectively Bflat) and its harmonics exactly in tune (right hand in proper position). When tuning a double horn both F and B-flat part must be tuned separately. Then the three (or six in case of a double horn) valve slides must be tuned separately in such a way that they exactly lower the horn a major second, a minor second and a minor third respectively. When the horn has been tuned in that way the simultaneous use of two valves will result in sharp tones. That residual mistuning can be corrected by the right hand bending or the embouchure. That last method is by far the most tiring and should be avoided. When tuning to an oboe a" only the main slide can be used because small adaptations on the main slide position will not much affect the overall horn tuning.

When playing a specific French horn over a long period of time its habits in mistuning are almost automatically corrected by right hand or embouchure. When tuning all slides of a new or other horn these automatic reflexes must be rejected very consciously and use a neutral embouchure (play the centre of the resonance peak) and hand position.

## Temperature

The resonance frequencies of the horn are dependent on the length of the horn tube and the speed  $v_{air}$  of sound, since the frequency  $f_t$  and the wavelength  $\lambda_t$  of the tone are related to the speed of sound as  $\lambda_t = v_{air} / f_t$ . The speed of sound in air is dependent on a number of variables like temperature, exact composition (and relative humidity) and pressure. Also the length of the horn is dependent on temperature: brass expands a little at rising temperatures. When comparing all these different parameters it turns out that the temperature of the air column inside the horn is the major factor of influence on the horn resonance frequencies. The blue graph in the picture here below describes the resonance frequencies (a'' at 25°C) of a horn at certain length and different temperatures. In green, red and yellow the frequencies of  $A_{\sharp}$ , A en  $A_{\flat}$  are depicted. Temperature changes of a few degrees have major influence on the actual resonance frequencies.



It is very difficult to play in tune at extreme conditions of temperature like in a cold church, outside in the freezing cold of changing the guard or in the sun of a tropical carnival parade (metal may reach a temperature of over 60°C in full sunlight). Even when the room temperature is well stabilised a perfect tuning is not obvious.

Air flowing out of the lungs has a temperature of up to 37°C. That air passes the (relatively cold) tube wall and is mixed with air already present inside the horn. The resulting temperature in the horn is dynamic, continuously changing, dependent on the speed of the air flow and local temperatures. Also an increasing or decreasing air flow, playing loud or soft, will affect the tuning. All these different temperatures together will ultimately result in some average that determines the resonance frequency. Furthermore the lips tend to change vibration frequency when playing louder or softer and a proper balance there is definitely not obvious. A sharp hearing and refined muscular motion control in embouchure and right hand use, are of crucial importance for a proper feedback of all these parameters.

When playing on a horn that is still cold, the tuning will be low initially and will rise to normal pitch during playing. To prevent a low pitch in the beginning of playing first the lips should be widened up a little (or placed around the rim of the mouthpiece) to let some warm air pass in a controlled way. After that the lips can be placed in normal playing position to begin your moving solo at much less need for immediate correction of the pitch. This method can be applied at most other instruments as well and is ever essential at extreme ambient temperatures.

#### The mouthpiece

Farquharson Cousins writes in his book "On playing the Horn" <sup>4)</sup> that "Blessed are they who start with a good mouthpiece and stay with it". Philip Farkas mentions that "An attempt to find the

perfect mouthpiece is definitely a pursuit of the will-o'-wisp"; choose a model that serves the different properties in a good (personal) balance. These properties are for instance:

- 1) comfort and ease of playing (in the different octaves)
- 2) sense to be in control (in the different octaves)
- 3) behaviour at loud and soft playing (in the different octaves)
- 4) tone quality (in the different octaves)
- 5) ease of lip trills
- 6) tuning in the high range

The mouthpiece properties have huge influence on the performance results. In reference <sup>7)</sup> calculated graphs are presented of; (graph a) the acoustical impedance (vertically in the characteristics) of a 2 meter long ( $D_i = 1$ cm) tube; (graph b) a neutrally loaded simple mouthpiece



and ; (graph c) mouthpiece and tube together. The frequency scale is horizontal from 0 to 1000 Hz. In graph (a) the resonances of the tube at 85 Hz and its harmonics are well recognizable. Graph (b) shows the broad resonance of the mouthpiece with a maximum at about 450 Hz. In graph (c) it is shown that when the mouthpiece is fitted to the tube, the resonances of the tube stay unmistakable. But in particular at the high frequency side of the mouthpiece resonance, the peak heights tend to

decrease fast. That means a fast drop in ease of playing at the higher resonance frequencies. Here an option would be a mouthpiece with a higher resonance frequency. That is dependent on mouthpiece volume and throat diameter. Unfortunately these parameters have as much influence on playability of the low resonances, tone quality, comfort of performance etc. etc. That makes a balancing of the options a very personal quest. Minute analysis of graph (c) shows that the properties of the mouthpiece affect the exact positions of the resonance peaks of this assembly, in particular in the high frequency range.

Difficult in putting mouthpieces to trial procedures is that in the process of testing, the embouchure is not yet fully developed for mastering that specific mouthpiece. Or a specific note can be cracked (in particular in the high range) because its resonance frequency was not conform the expected value. It is very time consuming to get fully adapted for a new mouthpiece. There is quite a number of different parameters to attend and when testing mouthpieces it is advisable to invite at least one other person for assessment, systematically follow the list of judged properties and register the findings for every mouthpiece.

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